Eur. Phys. J. B **49**, 103–108 (2006)
DOI: 10.1140/epjb/e2006-00029-9

The European Physical Journal B

Stochastic resonance and scale invariance in nonequilibrium metastable states

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Received 10 October 2005 / Received in final form 25 November 2005
Published online 31 January 2006 –

Abstract. Using computer simulations, we study metastability in a two-dimensional Ising ferromagnet relaxing toward a nonequilibrium steady state. The interplay between thermal and nonequilibrium fluctuations induces resonant and scale-invariant phenomena not observed in equilibrium. In particular, we measure noise-enhanced stability of the metastable state in a nonequilibrium environment. The limit of metastability, or pseudospinodal separating the metastable regime from the unstable one, exhibits reentrant behavior as a function of temperature for strong nonequilibrium conditions. Furthermore, when subject to both open boundaries and nonequilibrium fluctuations, the metastable system decays via well-defined avalanches. These exhibit power-law size and lifetime distributions, resembling the scale-free avalanche dynamics observed in real magnets and other complex systems. We expect some of these results to be verifiable in actual (impure) specimens.

PACS. 64.60.My metastable phases – 05.70.Ln Nonequilibrium and irreversible thermodynamics – 64.60.Qb nucleation – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion

1 Introduction and model definition

Many natural phenomena are characterized by the presence of metastable states slowing down the dynamics [1–4]. Metastability is a dynamic phenomenon not included in ensemble formalism [1], and its microscopic understanding still raises many fundamental questions. Studying simple models is therefore most useful. In particular, the two-dimensional Ising model has been subject to a number of analytical and numerical studies regarding metastability [1–11]. All these studies concern systems relaxing toward an equilibrium steady state. In this case the properties of the metastable state and its decay can be understood in terms of the system free energy functional. However, most interesting (and challenging) is the case in which the system converges toward a *nonequilibrium* stationary state [12]. Nonequilibrium conditions are more likely found in nature, and they characterize the evolution of most real systems. Far from equilibrium there is no free energy concept, and little is known on the properties of metastable states in this case. In particular, metastability in a nonequilibrium environment is relevant to the behavior of real magnetic particles, where dynamic impurities dominating the particle behavior cause break-up of detailed balance [6]. The aim of this paper is therefore to shed some light on the effects that a nonequilibrium perturbation has on the dynamic and static properties of metastable states in a simple system.

We structure the paper as follows. After introducing next our model and some of its known features, we study in Section 2 bulk metastability far from equilibrium. We find that the interplay between thermal and nonequilibrium noises induces stochastic resonance phenomena, as characterized by the mean lifetime of the metastable state and the pseudospinodal. Section 3 is devoted to explore the effect that open boundary conditions (a particular case of quenched disorder present in real nanoparticles) have on the system relaxation. In this case, the metastable system decays via scale-free avalanches which resemble the power-law avalanche dynamics observed in real magnets and other complex systems. The last section summarizes our conclusions.

Our model is one of the simplest *nonequilibrium* Ising ferromagnets. Consider a two-dimensional square lattice of side L with periodic boundary conditions. We define on each of its nodes $i \in [1, N \equiv L^2]$ a spin variable $s_i = \pm 1$. Spins interact among them and with an external magnetic field h via the Hamiltonian function $\mathcal{H} = -\sum' s_i s_j - h \sum_{i=1}^{N} s_i$, where the first sum runs over all nearest-neighbors pairs. In addition, we endow this

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model with a stochastic single spin-flip dynamics with transition rate,

$$\omega(\mathbf{s} \to \mathbf{s}^i) = p + (1-p)\frac{e^{-\beta \Delta \mathcal{H}_i}}{1+e^{-\beta \Delta \mathcal{H}_i}}, \qquad (1)$$

where $\mathbf{s}$ and $\mathbf{s}^i$ stand for the system configuration before and after flipping the spin at node i, respectively, $\Delta\mathcal{H}_i$ is the *energy* increment in such flip, and $\beta = 1/T$. In this paper we use a random sequential updating scheme. For any $0 < p < 1$ two different heat baths compete in (1): with probability p the spin flip is performed as if the system were in contact with an *infinite* temperature reservoir, while with probability $(1-p)$ the flip is performed at temperature T. Therefore, for $0 < p < 1$ a nonequilibrium steady state sets in asymptotically, characterized by a non-Gibbsian measure [12]. This is the simplest way of inducing nonequilibrium behavior in lattice systems [12], and it is assumed that this kind of stochastic, non−canonical perturbation for $p > 0$ may actually occur in real materials due to microscopic dynamic disorder and/or impurities, etc [6]. The zero-field model exhibits a second-order phase transition between a low-temperature ordered phase and a high-temperature disordered one [13]. This happens at a critical temperature $T_c(p) < T_c(p=0) \equiv T_{ons}$, where $T_{ons} \approx 2.2691$ is the Onsager temperature. Order disappears for $p > p_c \approx 0.17$ even for $T = 0$ [13]. On the other hand, for $p = 0$ we recover the usual *equilibrium* Ising model with Glauber dynamics.

2 Escape from metastable states and limit of metastability

For small $h < 0$ and $T < T_c(p)$, an initial system with all spins up, $s_i = +1$ $(i = 1,\ldots,N)$, quickly relaxes to a metastable state with positive magnetization, $m = N^{-1}\sum_{i=1}^{N} s_i > 0$. Such state eventually decays after a long time toward the true stable state, which corresponds now to $m < 0$. The relaxation time, also known as mean lifetime of the metastable state, $\tau(T,p,h)$, is a main feature characterizing the metastable system and its decay. More in detail, we define $\tau(T,p,h)$ as the mean first-passage time (in Monte Carlo steps per spin, MCSS) to $m = 0$. We measured this observable for values of T and p such that $T < T_c(p)$ and a magnetic field $h = -0.1$.

For intermediate and low temperature, the local stability of the metastable state is very strong, and the decay process is extremely slow, giving rise to mean lifetimes as large as 10^{40} MCSS (see Fig. 1). For this reason, simulations reported here required in practice using advanced Monte Carlo methods involving rejection-free techniques. In particular, we used the *s-1 Monte Carlo with absorbing Markov chains* (MCAMC) algorithm, and the *slow forcing approximation* [7,8]. The combination of both techniques has proven an invaluable tool for simulating systems with slow dynamics.

Figure 1 shows $\tau(T,p,h)$ as a function of T for $L = 53$ and $p \in [0, 0.01]$. Rather amazing, we observe

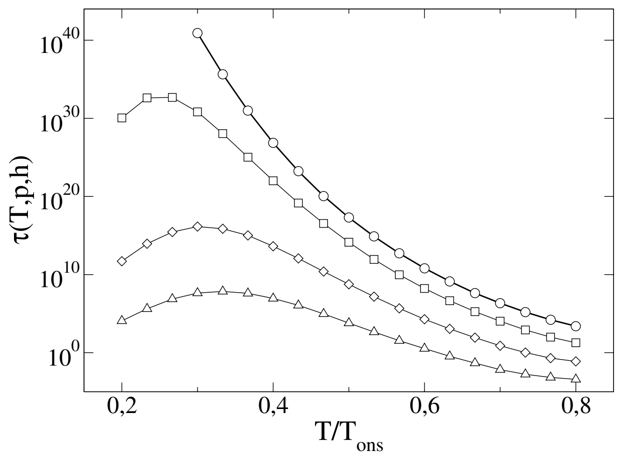

Fig. 1. Metastable lifetime results as a function of T for $L = 53$, $h = -0.1$ and, from top to bottom, $p = 0$, 0.001, 0.005 and 0.01. The nth curve (from top to bottom) is rescaled by a factor $10^{-2(n-1)}$. These results are obtained after averaging over $N_{exp} = 1000$ experiments.

that the nonequilibrium metastable lifetime exhibits nonmonotonous temperature dependence: $\tau(T,p,h)$ *increases* with temperature at low T for a fixed $p > 0$, showing a maximum at a non-trivial temperature $T_{max}(p)$, and then decreasing as temperature drops. That is, the local stability of the metastable state at low T and $p > 0$ (nonequilibrium environment) is enhanced by the addition of thermal noise. This behavior resembles the noise-enhanced stability (NES) phenomenon reported in experiments on unstable systems [14], and it is in contrast with the simple Arrhenius curve observed in equilibrium systems, i.e. $\tau \sim \exp(\Delta F/T)$, where ΔF is the free energy barrier associated to the equilibrium metastable state (see $p = 0$ in Fig. 1). Remarkably, adding *nonequilibrium noise* (increasing p) for a fixed T results always in a shorter τ, so only *thermal* NES is observed [15].

This stochastic resonance phenomenon suggests a nonlinear cooperative interplay between thermal and nonequilibrium fluctuations. That is, although both noise sources introduce disorder in the system when applied independently, their combined effect results in a resonant stabilization of the metastable state at low temperature.

One can further characterize metastable states far from equilibrium by studying the limit of metastability. When the magnetic field $|h|$ is increased, the *strength* of the metastable state decreases. That is, the local minimum in the potential energy landscape associated to the metastable state becomes less pronounced. Upon further increasing $|h|$, such local minimum eventually disappears. At this point the former metastable state becomes unstable: Its relaxation toward the final stable configuration is no longer hampered by any potential energy barrier. The magnetic field strength h^* at which the metastable-unstable transition occurs is called pseudospinodal [16], and we measure it in what follows.

To do so, we need a simple criterion to conclude that our model exhibits a metastable state for a given point

Table 1. Spin classes for the two-dimensional Ising model with periodic boundary conditions. The last column shows the energy cost of flipping the central spin.

Class	Central spin	Number of up neighbors	$\Delta\mathcal{H}$
1	+1	4	$8J + 2h$
2	+1	3	$4J + 2h$
3	+1	2	$2h$
4	+1	1	$-4J + 2h$
5	+1	0	$-8J + 2h$
6	−1	4	$-8J - 2h$
7	−1	3	$-4J - 2h$
8	−1	2	$-2h$
9	−1	1	$4J - 2h$
10	−1	0	$8J - 2h$

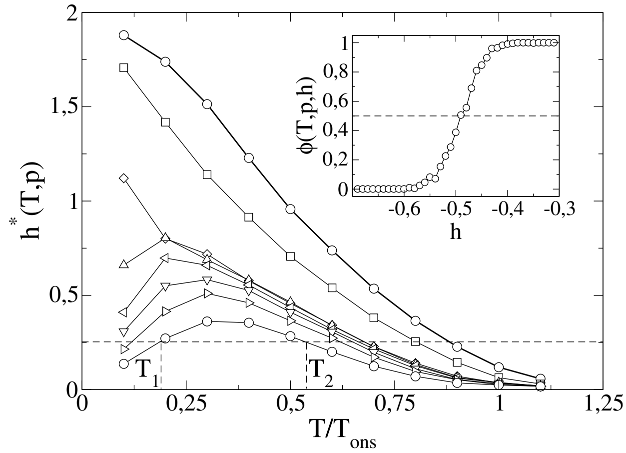

Fig. 2. Monte Carlo results for $h^*(T,p)$ as a function of T for $L = 53$ and, from top to bottom, $p = 0$, 0.01, 0.03, 0.0305, 0.0320, 0.0350, 0.04 and 0.05. Notice the change of asymptotic behavior in the low temperature limit for $p \in (0.03, 0.0305)$. Inset: The probability of the metastable state, as defined in the main text, as a function of $h < 0$ for $L = 53$, $T = 0.7T_{ons}$ and $p = 0$. Data here correspond to an average over 500 independent demagnetization experiments for each value of h. Error bars are smaller than the symbol sizes.

(T, p, h) in parameter space. Metastability is characterized in general by the presence of *free energy* barriers hampering the evolution toward the truly stable state. In this case, relaxation is an activated process controlled by large fluctuations. On the other hand, an unstable state decays without any hindrance. In both cases, for a given experiment j of a total of N_{exp} runs, relaxation from the initial state $\mathbf{s_1} \equiv \{s_i = +1, i = 1, \ldots, N\}$ will proceed through certain path in phase-space, $\sigma_j \equiv \{\mathbf{s_1}, \mathbf{s_2^{(j)}}, \ldots, \mathbf{s_{\Gamma(j)}^{(j)}}\}$. Here $\mathbf{s_l^{(j)}}$ is the l-th configuration, starting from $\mathbf{s_1}$, of a total number of $\Gamma(j)$ configurations which make up the path in experiment j. At any stage $\mathbf{s_l}^{(j)}$ of this path, we can define (see below) a net tendency $\Lambda(\mathbf{s_l}^{(j)})$ of the system to evolve toward the stable state. A state $\mathbf{s_l}^{(j)}$ such that $\Lambda(\mathbf{s_l}^{(j)}) < 0$ belongs to the metastable basin. In this way, we may divide relaxation paths in two different types. On one hand, *metastable paths*, in which at least one configuration $\mathbf{s_l^{(j)}} \in \sigma_j$ exists such that $\Lambda(\mathbf{s_l^{(j)}}) < 0$, and on the other hand, *unstable paths*, such that $\Lambda(\mathbf{s_l^{(j)}}) > 0 \; \forall \mathbf{s_l^{(j)}} \in \sigma_j$ (in both cases we exclude the final stable configuration).

The function $\Lambda(\mathbf{s})$ can be defined by noting that any spin in the system can be associated to a unique *spin class*, defined by the spin state, $s = \pm 1$, and the number of its up nearest neighbors, $n \in [0, 4]$. For periodic boundary conditions, there are 10 different *spin classes*, as shown in Table 1. The energy cost $\Delta\mathcal{H}(s, n)$ of flipping any spin within a class is the same. That is, the rate (1) only depends on s and n, which define the class. Now, if $n_k(\mathbf{s})$ is the number of spins in class k when the system is in configuration $\mathbf{s}$, and $\omega_k \equiv \omega(s, n)$ is the transition rate for this class, the function

$$G(\mathbf{s}) = \sum_{k=1}^{5} n_k(\mathbf{s})\omega_k. \quad (2)$$

gives the number of up spins which flip per unit time in state $\mathbf{s}$ (note that classes $k \in [1, 5]$ correspond to a central up spin, see Tab. 1). Since $h < 0$, $G(\mathbf{s})$ is the growth rate of the stable phase. Similarly [8], the stable phase shrinkage rate is given by $S(\mathbf{s}) = \sum_{k=6}^{10} n_k(\mathbf{s})\omega_k$. The rates $G(\mathbf{s})$ and $S(\mathbf{s})$ determine the tendency of the system to evolve toward the stable or metastable states at a given phase-space point $\mathbf{s}$, respectively. Therefore we may write $\Lambda(\mathbf{s}) \equiv G(\mathbf{s}) - S(\mathbf{s})$.

For fixed T, p and $h < 0$, given the stochasticity of the dynamics, one needs to be concerned with the *probability* of occurrence of metastability, defined as $\phi(T, p, h) \equiv n_{met}/N_{exp}$, where $n_{met}(T, p, h)$ is the number of experiments out of the total N_{exp} in which the relaxation path in phase space belongs to the class of *metastable paths*. This is shown in the inset to Figure 2. The limit of metastability or pseudospinodal field, $h^*(T, p)$, is defined in this scheme as the field for which $\phi(T, p, h^*) = 0.5$; this is shown in Figure 2 for a system with $L = 53$ [17].

Remarkably, we find an instability separating two different low-temperature behaviors for $h^*(T, p)$, depending on the amplitude of nonequilibrium fluctuations. For small enough values of p, namely, $p \in [0, 0.03]$, which includes the equilibrium case, $p = 0$, the field $h^*(T, p)$ monotonously grows and extrapolates to 2 as $T \to 0$. For larger p, namely, $p \in [0.0305, 0.17)$, however, $h^*(T, p) \to 0$ in the low temperature limit, exhibiting a non-trivial maximum at intermediate T. The value $p = \pi_c \approx 0.03025$ separates the two types of asymptotic behavior.

The $p < \pi_c$ regime can be understood on simple grounds. In this case, $h^*(T, p)$ increases as T drops, meaning that we need a stronger field to *destroy* metastability as T decreases. In a metastable state, the system tendency to maintain spin order competes with and overcomes the tendency of the individuals spins to line up with the external magnetic field. Since both T and p induce disorder, one naively expects that decreasing T and/or p (i.e. increasing order), a stronger magnetic field will be needed to destroy the metastable state, as confirmed in Figure 2 for $p < \pi_c$.

On the other hand, the behavior for $p > \pi_c$ is more intriguing. Consider for instance the case $p = 0.05 > \pi_c$ and $|h| = 0.25$. According to Figure 2, we can define two different temperatures, $T_1 < T_2$, such that metastability is only observed for $T \in [T_1, T_2]$. The fact that h^* goes to zero as $T \rightarrow 0$ for $p > \pi_c$ means that, at low temperature, the nonequilibrium fluctuations parameterized by p are strong enough to *destroy* on their own the metastable state. Based on the above naive argument, one would expect that increasing T adds disorder to the system, so no metastable states should in principle show up. However, we observe metastability for intermediate temperatures, $T \in [T_1, T_2]$. This reentrant behavior of the pseudospinodal field suggests once more a resonant interplay between thermal and nonequilibrium fluctuations in the system: although both T and p add independently disorder to the system, their nonlinear interplay determine the existence of regions in parameter space (T, p, h) in which no metastable states are observed at low and high temperature, but instead emerge at intermediate T.

Both phenomena reported here, i.e. the non-monotonous T-dependence of the metastable-state mean lifetime and the reentrant behavior of the pseudospinodal field, are expressions of an underlying stochastic resonance phenomenon. Interesting enough, however, they present an essential difference. That is, while the resonant stabilization of the metastable state lifetime (a dynamic phenomenon) is observed for any $p > 0$, the reentrant behavior of the pseudospinodal field (a thermodynamic observable) only emerges for *strong* nonequilibrium conditions, $p > \pi_c$. The origin of this difference is not clear.

3 Avalanches during relaxation

The previous observations concern the *bulk* metastability. However, in real magnets, one needs in practice to create and to control fine grains, i.e., magnetic particles with *borders* whose size ranges from mesoscopic to atomic levels, namely, clusters of 10^4 to 10^2 spins, and even smaller ones. Though experimental techniques are already accurate for the purpose, the underlying physics is much less understood than for bulk properties. In particular, one cannot assume that such particles are neither *infinite* nor *pure*. That is, they have free borders, which results in a large surface/volume ratio inducing strong border effects, and impurities. Motivated by the experimental situation, we also studied a finite two−dimensional system subject to open circular boundary conditions. The system is defined on a square lattice, where we inscribe a circle of radius r; sites outside this circle do not belong to the system (their bonds to spins in the system are broken). We mainly report in the following on a set of fixed values for the model parameters, namely, $h = -0.1$, $T = 0.11\,T_{ons}$ and $p = 10^{-6}$. This choice is dictated by simplicity and also because (after exploring the behavior for other cases) we came to the conclusion that this corresponds to an interesting region of the system parameter space, where: (i) many metastable states emerge slowing down the system relaxation, (ii) the effects of p and T are comparable, and (iii) clusters are compact and hence easy to analyze.

The effects of free borders on the metastable-stable transition have already been studied for equilibrium systems [11]. In this case, the system evolves to the stable state through the *heterogeneous* nucleation of one or several critical droplets which always appear at the system's border. That is, the free border acts as a droplet condenser. This is so because it is energetically favorable for the droplet to nucleate at the border. Apart from this, the properties of the metastable-stable transition in equilibrium ferromagnetic nanoparticles do not change qualitatively as compared to the periodic boundary conditions case [11]. In our nonequilibrium system we observe similar behavior. However, the structured fluctuations that the nonequilibrium metastable system shows as it evolves towards the stable state subject to the combined action of free borders and the nonequilibrium perturbation are quite unexpected.

As illustrated in Figure 3a, the relaxation of magnetization occurs via a sequence of well−defined abrupt jumps. That is, when the system relaxation is observed after each MCSS, which corresponds to a 'macroscopic' time scale, strictly monotonic changes of $m(t)$ can be identified that we shall call *avalanches* in the following. To be precise, consider the avalanche beginning at time t_a, when the system magnetization is $m(t_a)$, and finishing at t_b. We define its *size* and *duration*, respectively, as $\Delta_m = |m(t_b) - m(t_a)|$ and $\Delta_t = |t_b - t_a|$. Our interest is on the histograms $P(\Delta_m)$ and $P(\Delta_t)$. Figure 3b shows the large avalanche size distribution $P(\Delta_m)$ for particle sizes $r = 30$, 42, 60, 84 and 120, after an extrinsic noise [18] (i.e. some trivial, exponentially distributed small fluctuations apparent in the magnetization plateaus in the inset to Fig. 3a) has been subtracted. A power law behavior is clearly observed. The measured power law exponents, $P(\Delta_m) \sim \Delta_m^{-\eta(r)}$, show size-dependent corrections to scaling of the form $\eta(r) = \eta_\infty + a_1 r^{-2}$, with $\eta_\infty = 1.71(4)$. The duration distribution also exhibits power law behavior, $P(\Delta_t) \sim \Delta_t^{-\alpha(r)}$, and again $\alpha(r) = \alpha_\infty + a_2 r^{-2}$ with $\alpha_\infty = 2.25(3)$. In both case, size and duration, the power law ends with an exponential cutoff, $P(\Delta) \sim \exp(-\Delta/\bar{\Delta})$. Remarkably, size and duration cutoffs also scale algebraically with system size, $\bar{\Delta} \sim r^\beta$, with $\beta_m = 2.32(6)$ and $\beta_t = 1.53(3)$. Similar finite size corrections to exponents and cutoffs have been also found in experimental systems [19,20]. The observed power laws imply that avalanches are scale-free (up to certain maximum size and duration) in our nonequilibrium ferromagnet subjected to open boundary conditions. We also measured avalanches for $p = 0$ in the circular magnetic particle, and for $p \neq 0$ in the periodic boundary conditions system. In both cases only small avalanches occur and the distributions are exponential-like, thus indicating the absence of scale invariance. That is, the combined action of free boundaries and nonequilibrium impurities is behind the large, scale-free avalanches, and essentially differs from the standard bulk noise driving the system and causing small, exponentially distributed avalanches.

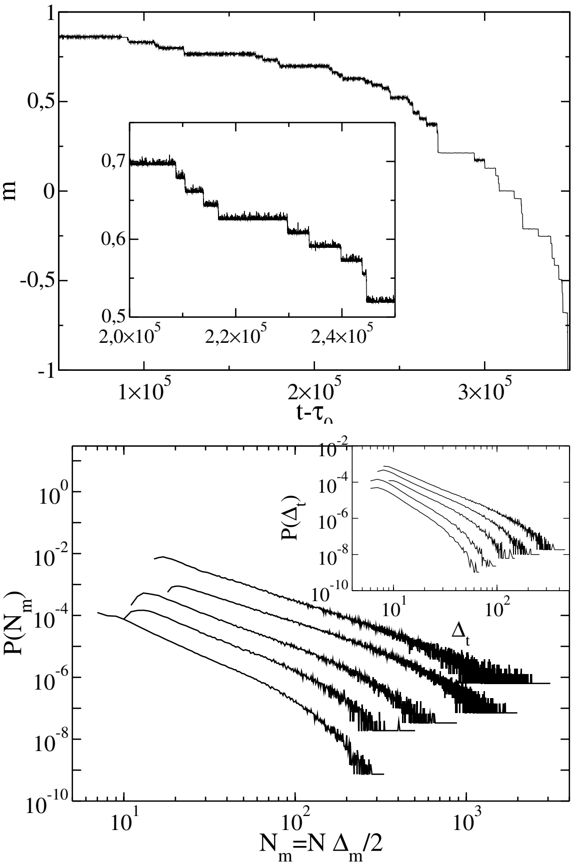

Fig. 3. (a) Time variation of the magnetization showing the decay from a metastable state for a $r = 30$ particle; avalanches are seen here by direct inspection. Time is in Monte Carlo Steps per Spin (MCSS), and $\tau_0 \sim 10^{30}$ MCSS. The inset shows a detail of the relaxation. (b) Large avalanche size distribution $P(\Delta_m)$ for the circular magnetic nanoparticle and, from bottom to top, $r = 30$, 42, 60, 84 and 120. (For visual convenience, curves have been shifted vertically.) The inset shows the avalanche duration distribution for the same system sizes.

Scale-invariant (or $1/f$) noise has been found in many complex systems, ranging from electronic devices to superconducting vortices, human cognition, earthquakes, and radiation from white dwarfs, to mention some. The hypothesis that some underlying mechanism is common to many situations is therefore appealing. Many possible generic mechanisms have been proposed in literature, most of them based on possible underlying critical phenomena. After much effort, there is no full agreement on a globally coherent explanation, however. We have shown above that our simple model of metastable magnetic particle exhibits scale-free avalanches during the decay process. Remarkably, the properties of these structured fluctuations in our oversimplified model are indistinguishable in practice from what one measures in many natural phenomena showing $1/f$ noise. For instance, size corrections similar to the ones observed here for $\eta(r)$ and $\alpha(r)$ have been reported in avalanche experiments on rice piles [19], and our values for η_∞ and α_∞ are amazingly close to those reported in some magnetic experiments for *quasi-two dimensional systems* [18]. Moreover, our cutoff values follow the precise trend observed, for instance, in magnetic materials. All these facts indicate that our simple model may contain the fundamental ingredients characterizing a whole class of natural systems in which a series of transitions between many different, short-lived metastable states characterize the dynamics (see Fig. 3a). Most interesting, we tested the presence of an underlying critical point responsible of the observed scale-invariant avalanches, with negative results (no divergent correlation time and/or length scales were found). Therefore, a non-critical dynamic mechanism seems to be responsible of this complex behavior, and preliminary studies [10] show that a superposition of many different scales that emerges due to the interplay of nonequilibrium fluctuations and free borders might give rise to the observed scale-free avalanches.

4 Conclusion

We have studied in this paper the escape from metastable states in a two-dimensional Ising ferromagnet evolving toward a nonequilibrium steady state. Relaxation in this case is considerably enriched as compared to equilibrium. In particular, we have shown that the stability of the metastable state at low T is enhanced by the addition of thermal noise. In particular, the mean lifetime of the metastable state has a peak as a function of T for any $p > 0$ (nonequilibrium conditions). This resonant stabilization of the metastable state is reminiscent of the noise-enhanced stability phenomenon measured in experiments with unstable systems [14], and it is in contrast with the simple Arrhenius curve observed in equilibrium systems.

In addition, the limit of metastability or pseudospinodal magnetic field $h^*(T, p)$, that separates the metastable and unstable regions in parameter space, exhibits reentrant behavior as a function of temperature for strong nonequilibrium conditions. That is, for $p > \pi_c \approx 0.03025$, metastability is not observed for low and high temperature, but instead emerges for intermediate temperatures.

These results point out to an underlying stochastic resonance phenomenon due to the nonlinear cooperative interplay between thermal and nonequilibrium fluctuations. That is, although both noise sources introduce independently disorder in the system, their combined effect results in higher levels of order at low temperature, as deduced from the resonant stabilization of $\tau(T, p, h)$ and the reentrant behavior of $h^*(T, p, h)$.

We also observe that, under the action of both the nonequilibrium impurity and free borders, the metastable-stable transition proceeds by avalanches. These are power-law distributed, up to certain size-dependent cutoffs. We do not detect, however, any underlying critical point responsible of the observed scale invariance. This fact, together with the striking similarities between the statistical properties of the structured fluctuations in our model and those of avalanches observed in many real complex systems, suggests a non-critical common mechanism responsible of the ubiquitously observed $1/f$ noise in systems characterized by a rich, varied set of transitions between many short-lived metastable states.

A theoretical understanding of the observed behavior seems challenging. The nonequilibrium character of our model prevents in principle any formulation of the metastable problem in terms of free-energy functions controlling the nucleation of stable-phase droplets inside the metastable sea. Understanding the macroscopic potential characterizing the complex collective behavior observed in this far-from-equilibrium model is a major open challenge.

We acknowledge financial support under the Spanish project MEyC FIS2005-00791. PIH also acknowledges the Spanish MEyC for continuous support.

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15. For parameters (T, p, h) such that the mean lifetime is not very large, one can measure $\tau(T, p, h)$ using standard Monte Carlo methods. In all cases, results obtained with standard and rejection-free techniques agree perfecly. In particular, the noise-enhanced stability (NES) phenomenon is recovered in standard simulations, ruling out the possibility of NES being an artifact due to the slow-forcing approximation.
16. The prefix *pseudo* in pseudospinodal stems from the fact that the metastable-unstable transition is not a sharp transition at h^*, but instead it is a progressive crossover from a metastable phase for $|h| < h^*$ to an unstable one for $|h| > h^*$ (see inset to Fig. 2).
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